FINAL REPORT

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MATHEMATICAL PROBLEMS OF NONLINEAR WAVE PROPAGATION AND OF WAVES IN HETEROGENEOUS MEDIA

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I. BRIEF OUTLINE OF RESEARCH FINDINGS

The research findings under this grant are contained in the research papers listed in Section II of this report. Some of them have been published already, others have been submitted for publication and accepted, and others have not yet been accepted. The status of each paper is indicated after its title. Now we shall mention some of the findings explicitly.

Professor Keller and Dr. Givoli extended their work on non-reflecting boundary conditions to elastic waves. They also showed how to modify this work to develop a finite element method for domains with corners.

Professors Keller and Ting developed a theory for the motion of slender jets and thin sheets taking account of surface tension.

Professors Keller and Knessl analyzed the stability of rotating shear flows in shallow water by an asymptotic method.

Professors Keller and Ting treated singularities of semilinear waves. They showed that a remarkable collection of singularities can emerge from the collision of a finite number of shocks.

Professors Keller, Rubinstein and Sternberg analyzed front interaction for tri- stable reaction diffusion equations.

Professor Keller and Lowengrub developed asymptotic and numerical methods for solving the semilinear heat equation when its solutions blow up.

II. MANUSCRIPTS PUBLISHED OR SUBMITTED FOR PUBLICATION

1. Non-reflecting boundary conditions for elastic waves (with D. Givoli), *Wave Motion*, 12, 261–279, 1990.

2. Collapse of wavefunctions and probability densities, *Am. J. Phys.*, 58, 768–770, 1990.

3. Slender jets and thin sheets with surface tension (with L. Ting), *SIAM J. Appl Math.*, 50, 1533–1546, 1990.

4. Stirling number asymptotics from recursion equations using the ray method, (with C. Knessl), *Studies in Applied Math.* 84, 43–56, 1991.

5. Diffusively coupled dynamical systems, *Applied and Industrial Mathematics*, R. Spigler, ed., Kluwer, Amsterdam 1991, 49–56.

6. Changes in adiabatic invariants (with Ye Mu), *Annals of Physics*, **205**, 219–227, 1991.

7. Asymptotic properties of eigenvalues of integral equations, (with C. Knessl), *SIAM J. Appl. Math.*, **51**, 214–232, 1991.

8. Nonlinear wave motion in a strong potential, (with J. Rubinstein), *Wave Motion*, **13**, 291–302, 1991.

9. Nonlinear eigenvalue problems under strong localized perturbations with applications to chemical reactors, (with M.J. Ward), *Studies in Applied Math.* **85**, 1–28, 1991.

10. Mathematical model of granulocytopoiesis and chronic myelogenous leukemia, (with A.S. Fokas and B.D. Clarkson), *Cancer Research*, **51**, 2084–2091, 1991.

11. Flexural rigidity of a liquid surface, (with G.J. Merchant), *J. Statistical Phys.*, **63**, 1039–1051, 1991.

12. Family data determine all parameters in Mendelian incomplete penetrance models (with A.S. Whittemore and M.J. Ward), *Ann. Hum. Genet.* **55**, 175–177, 1991.

13. Low-Grade, Latent Prostate Cancer Volume: Predictor of Clinical Cancer Incidence?, (with A.S. Whittemore and R. Betensky), *J. Natl. Cancer Inst.*, **83**, 1231–1235, 1991.

14. Asymptotic behavior of high order differences of the partition function, (with C. Knessl), *Comm. Pure Appl. Math.*, **44**, 1033– 1045, 1991.

15. Surface tension (with A. King and G.J. Merchant), *Of Fluid Mechanics and Related Matters, Proceedings of a Symposium Honoring John Miles on his Seventieth Birthday*, R. Salmon and D. Betts, eds., Scripps Institute of Oceanography, U.C.S.D., 1991, 161–168.

16. Free surface flow around a ship, (with J.-M. Vanden-Broeck), *Mathematical Approaches in Hydrodynamics*, Touvia Miloh, ed., SIAM, Philadelphia, 1991, 289–299.

17. Contact Angles, (with G.J. Merchant), *Phys. Fluids A* 4, 477–485, 1992.

18. A finite element method for domains with corners, (with D. Givoli and L. Rivkin), *Int. J. Num. Meth. Eng.*, **35**, 1329–1345, 1992.

19. Stability of rotating shear flows in shallow water, (with C. Knessl), *J. Fluid Mech.*, **244**, 605–614, 1992.

20. The shape of a Möbius band, (with L. Mahadevan), *Proc. Roy. Soc. Lond. A*, **440**, 149–162, 1993.

21. Drop evaporation through a thin membrane, (with H.L. Frisch), *J. Colloid and Interface Science,* **155**, 262–263, 1993.

22. Phase fronts in reaction-diffusion problems, *Emerging applications in free boundary problems*, J. Chadam and H. Rasmussen, eds., John Wiley & Sons, Inc., New York, 24–28, 1993.

23. Singularities of semilinear waves, (with L. Ting), *Comm. Pure Appl. Math.*, **46**, 341–352, 1993.

24. The stability of growing or evaporating crystals, (with R. Ghez and H.G. Cohen), *J. Appl. Phys.*, **73**, 3685–3693, 1993.

25. The stability of rapidly growing or evaporating crystals, (with G.J. Merchant and H.G. Cohen), *J. Appl. Phys.*, **73**, 3694–3697, 1993.

26. Strong localized perturbations of eigenvalue problems, (with M. J. Ward), *SIAM J. Appl. Math.*, **53**, 770–798, 1993.

27. Summing logarithmic expansions for singularly perturbed eigenvalue problems, (with M.J. Ward and W.D. Henshaw), *SIAM J. Appl. Math.* **53**, 799–828, 1993.

28. Stresses in narrow regions, *J. Appl. Mech.*, (accepted 9/92).

29. Front interaction and nonhomogeneous equilibria for tri-stable reaction-diffusion equations, (with J. Rubinstein and P. Sternberg), *SIAM J. Appl. Math.*, **53**, 1669–1685, 1993.

30. Accurate procedures for approximate Bayesian and conditional inference without the need for orthogonal parameters, (with T.J. DiCiccio and M.A. Martin), *Biometrika*, (submitted 2/92, revised ?/92).

31. Asymptotic and numerical results for blowing-up solutions to semilinear heat equations, (with J. Lowengrub), *Proc. NATO Conference on Singularities*, ed. Russell Caflisch, Crete, 1992.

32. Asymptotic evaluation of oscillatory sums, (with C. Knessl), *Eur. J. Appl. Math.*, **4**, 361–380, 1993.

33. Asymptotic methods for partial differential equations: the reduced wave equation and Maxwell's equations, (with R.M. Lewis), *Surveys in Applied Mathematics*, edited by J.B. Keller, G. Papanicolau, and D. McLaughlin, Plenum Publishing, NY, 1993, (accepted).

34. A Characterization of the Poisson Distribution and the Probability of Winning a Game, *Am. Stat.*, (submitted 8/92).

35. Tension and thick filament movement in muscle fibers, (with G.J. Merchant), *Biophysical J.*, (submitted 6/92).

36. Eulerian number asymptotics (with E. Giladi). *Proc. Royal Soc.*, (sub 1/22/93).

37. Removing small features from computational domains, *J. of Computational Phys.*, (sub 8/92).